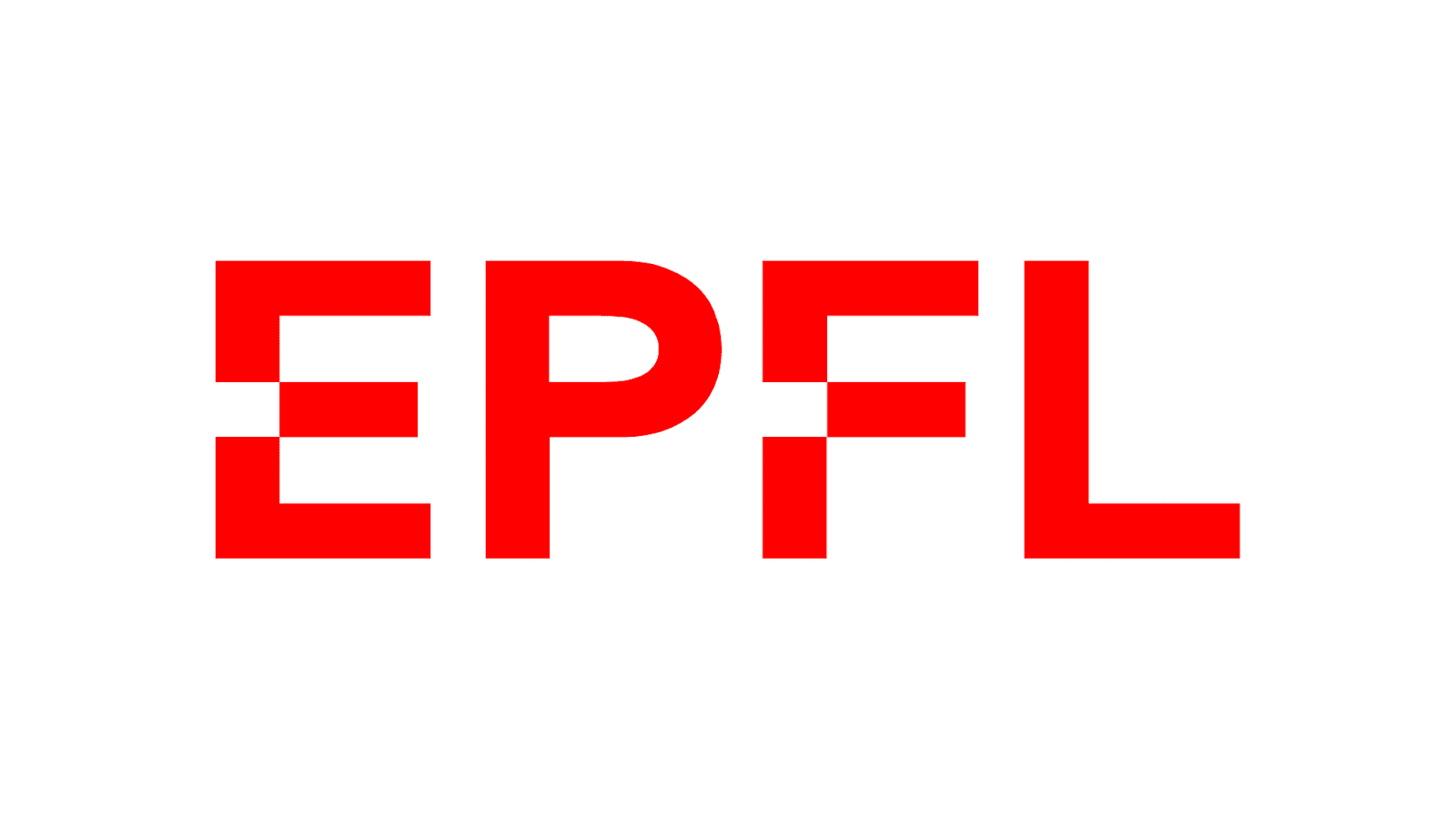
**École Polytechnique Fédérale de Lausanne**



Wireless receivers: algorithms and architectures

**OFDM Project**

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# Introduction

Insingle-carrier modulation, all data is transmitted using one carrier frequency, making it simple but highly sensitive to channel impairments like multipath fading and interference.

This was not a reliable modulation for modern technologies where we seek reliability and security. So that’s why we avert to multi-carrier modulation techniques. The idea is to divide the data across multiple carrier frequencies and send them with N parallel streams, improving resilience to impairments.

Orthogonal Frequency Division Multiplexing (OFDM) is a cornerstone modulation technique in modern digital communications, celebrated for its capacity to efficiently utilize bandwidth while mitigating intersymbol interference (ISI) and combating the challenges of multipath fading. As communication systems demand greater data rates and robustness, OFDM has emerged as an optimal solution, striking a balance between spectral efficiency and system reliability.

OFDM divides the available channel bandwidth into numerous subcarriers, each transmitting data independently. These subcarriers, spaced at precise intervals based on the symbol rate, maintain orthogonality in the frequency domain. This orthogonality ensures that signals transmitted over one subcarrier do not interfere with those on adjacent subcarriers, thereby upholding the principles of interference-free transmission while enabling simultaneous data streams.

One of the most compelling advantages of OFDM lies in its simplified channel equalization. The narrowband nature of each subcarrier ensures that the channel response remains nearly constant within its scope, reducing the complexity of combating multipath effects. This trait, combined with OFDM's inherent resilience to noise and interference, renders it particularly effective in challenging environments with significant distortion or reflection.

This report delves into the theoretical foundation of OFDM, tracing its evolution from single-carrier modulation to its current form as a multicarrier paradigm. Through an exploration of system design and implementation, the interplay between critical design parameters and performance metrics such as Bit Error Rate (BER) and channel spectrum are analysed. The discussion also includes various tracking methods to address channel variability, ensuring a comprehensive understanding of the principles and practicalities of OFDM communication systems.

A diagram of different colors

Description automatically generated

Figure 1 Single vs. Multi carrier comparison

The signal after the modulation will have the following format:

Where X(k) is the data stream transmitted on the k-th subcarrier, T is the duration of the OFDM signal, is the frequency of the k-th subcarrier, where is the subcarrier spacing with .

With a sampling interval , we will get to the sampled version of the OFDM signal, given by:

The last equation shows the expression of the Inverse Discrete Fourier Transform (IDFT), showing that IFFT modulation can be used.

# Transmitting the signal

We have some raw bits that need to be transmitted. For this to be done we need a clear plan of the steps we need to take to process the bits into a signal suitable for transmission through the physical channel. For that we built the following block diagram and followed it up with a suitable Matlab code.

A diagram of a computer program

Description automatically generated with medium confidence

Figure 2 Block diagram of transmitter

## Mapping

The first step that need to be taken is finding a way to map them into symbols.

In digital communication systems, information is represented in bits (binary data). However, transmission over a physical channel is achieved using modulated waveforms, not raw binary sequences. Mapping bits into symbols is an essential step to bridge the digital world (bits) with the analog domain (waveforms). Symbols represent groups of bits as distinct values or points in a constellation diagram, each corresponding to a unique waveform. In this project Gray QPSK mapping will be used. QPSK is a digital modulation scheme where two bits are mapped onto a single symbol, resulting in four unique symbols. Each symbol corresponds to a phase shift in the carrier signal, typically 0, 90, 180 or 270 degrees. Each constellation point is equidistant from the origin to represent equal energy, and the angle of each point corresponds to a specific phase of the carrier. The Gray part comes into play so that we ensure the symbols differ by 1 bit from one another.

## Forming the TX symbols

The next step is to combine the bits we are transmitting with the training symbols sequence. The training symbols are predefined sequences of bits known to both the transmitter and receiver. They play a crucial role in channel estimation and equalization, enabling the system to mitigate the distortion effects caused by the real channel.  
Two different ways of generating the training symbols were used:   
**block-based** and **comb-based** training methods.

### Block-based training

In the block-based approach, training symbols are transmitted periodically in an OFDM system, with each training symbol spanning across all the subcarriers in the system. These periodic training symbols enable the receiver to perform channel estimation at regular intervals, allowing it to adapt to slowly varying channel conditions effectively.

* Training placement – training sequence T, repeated periodically every K symbols:

, K – block interval, – data symbols

* Channel estimation – the received training symbols are related to the transmitted training symbols by the channel resposnse and the noise:
* Data equalization – for every subsequent data block we equalize with the channel response:

### Comb-based training

On the other hand, the comb-based technique distributes training symbols across the subcarriers within each symbol, with a chess-like structure. By embedding training information within every transmission, the comb-based method supports real-time channel estimation for scenarios with rapid channel variations. The subcarriers are divided into pilot subcarriers, which will be used for channel estimation, and data subcarriers, for the payload.

At a specific subcarrier Pm, where m indexes the pilot subcarriers we have the following:

Since is know, can be estimated as:

To estimate the channel response for non-pilot subcarriers we need to use interpolation techniques. In this case, linear interpolation is used, as following:

While block-based training conserves bandwidth by using fewer training symbols, comb-based training enhances the system's ability to track fast changes in the channel, offering a trade-off between spectral efficiency and the ability to combat fading and distortion.

A comparison of different types of symbols

Description automatically generated

Figure 3 Block and Comb training methods

## Serial to Parallel Conversion

Next logical step is converting the serial signal to parallel N streams to mimic the nature of OFDM. A function is used that reshapes the data into 2D matrix, where the number of rows represents the number of subcarriers, and the columns represent the data for the OFDM symbols distributed across the subcarriers. Each subcarrier’s data is in a separate row, giving the parallel look.

## Inverse Fast Fourier Transform

This operation is used to transform the data from frequency to the time domain, so that the signal can be physically transmitted through the channel. It’s done by combining the modulated subcarriers into a single composite time-domain signal, while ensuring the orthogonality of the subcarriers, to minimize the interference between them.

## Adding Cyclic Prefix

The cyclic prefix (CP) is added to each OFDM symbol by copying the last ​ samples of the symbol and appending them to its beginning. This ensures that the transmitted signal experiences circular convolution with the channel, enabling efficient equalization in the frequency domain using FFT. The CP also acts as a guard interval to combat Inter-Symbol Interference (ISI) caused by multipath fading, preventing overlapping symbols from interfering with one another.

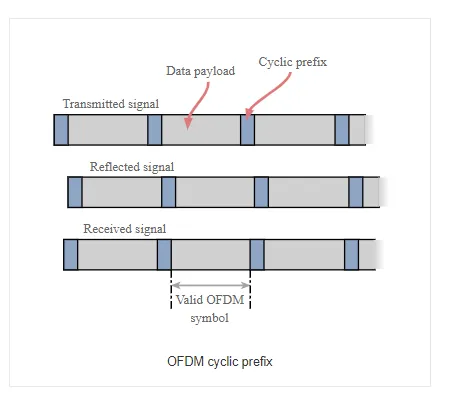


Figure 4 Representation of Cyclic Prefix positioning

Mathematically, for an OFDM symbol x[n] of length N, the transmitted symbol with CP is:

Important note to add is that the cyclic prefix length ​ must be grater than the maximum channel delay spread to fully mitigate the ISI. While adding the CP reduces spectral efficiency, it is crucial for preserving subcarrier orthogonality and ensuring reliable communication in multipath environments.

## Parallel to serial

We need to get the signal to the original serial form because it needs to pass through the actual physical channel which supports serial data.

## Preamble

On the other hand, there are some processing steps done on the preamble as well, which is essential for synchronization. The preamble is generated like a random like bit sequence. It needs to be mapped into symbols, so in this case BPSK mapping is used. Each bit is represented by one of 2 phases: 0 or π. Bit value of 0 is mapped into +1, and a bit value of 1 is mapped into -1. Then the preamble is up sampled to match the transmission rate of the transmission signal and for better precision to be achieved when syncing. At the end the preamble is pulse shaped with the RRC filter so that the bandwidth and the ISI are limited.

## Normalizing signal energy

After inserting the preamble at the start of the signal, the energy of the signal is normalized. Since QPSK mapping is used, normalizing the energy does not cause issues with ISI, as the constellation points remain well-separated. However, with higher-order modulation schemes like 16-QAM or 32-QAM, normalization can compress the constellation points, potentially increasing susceptibility to noise and ISI.

## Up conversion

At the very end before transmitting the signal though the channel up conversion is done, where the baseband signal is shifted to higher frequency, in particular the carrier frequency .

A screen shot of a sound wave

Description automatically generated

Figure 5 Signal that was transmitted

A screen shot of a graph

Description automatically generated

Figure 6 Spectrum of the transmitted signal

# Receiving the signal

After the signal had passed through the physical channel, on the receiver side some processing steps need to be done. The following block diagram represent the steps taken in this project:

A diagram of a flowchart

Description automatically generated

Figure 7 Block diagram of receiver process flow

Firstly, the received bits are now affected by the channel and its impairments, like fading, and noise.

## Down conversion

To process the received signal, down-conversion is performed to shift the signal from the carrier frequency back to the baseband. At the receiver, the incoming radio frequency (RF) signal is mixed with a local oscillator (LO) signal at the carrier frequency ​, effectively removing the high-frequency carrier. This operation isolates the original baseband signal, which contains the transmitted data.

Mathematically, if the received RF signal is:

When mixed with the LO signal it will produce the following:

The first term represents the baseband signal, while the second one is the high frequency part that needs to be removed, which is done in the next step.

## Low Pass Filtering

Since the channel in OFDM modulation acts like a filter, LP Filter will be enough to filter out the high frequencies. This means that the signal will be cut just above the bandwidth, to avoid removing important signal information. In this project we cut off at 30dB, meaning the signal will be attenuated to of the original amplitude. With this we ensure that we remove the noise and successfully isolate the baseband signal for further processing.

## Frame synchronization

To determine where the actual signal starts, frame synchronization needs to be performed. The start of the data is detected by cross correlating the received signal with the known preamble. The matched filter output, combined with a detection threshold, identifies the peak correlation point, which corresponds to the beginning of the data frame. The synchronization process ensures accurate alignment of the receiver with the transmitted signal, compensating for timing offsets introduced by the channel.

## Cyclic Prefix Removal

The next logical step is to remove the CP to extract the payload and training symbols. This ensures that the remaining data corresponds exactly to one OFDM symbol, ready for further operations like FFT and demodulation, without contamination from ISI.

## Serial to Parallel Conversion

Serial-to-parallel conversion is necessary before performing the FFT in an OFDM system because the FFT operates on parallel data streams corresponding to individual subcarriers. The received signal is a serial stream, but each OFDM symbol consists of multiple subcarrier signals transmitted simultaneously. By converting the serial stream into parallel data, each subcarrier's time-domain samples are correctly aligned, enabling the FFT to efficiently transform the data into the frequency domain for demodulation.

## Fast Fourier Transform

FFT makes sure the signal streams are correctly converted to the frequency domain from the time domain. FFT decomposes the dime domain discrete signal with length N into frequency components producing the following discrete signal:

This operation reveals the amplitude and phase of each frequency component k in the signal, corresponding to the subcarriers in OFDM.

What is done essentially is that FFT maps the received signal onto its respective subcarriers, effectively demultiplexing the data. Since OFDM modulates data across orthogonal subcarriers, the FFT ensures that the subcarriers remain independent, and their data symbols are efficiently extracted for demodulation and decoding.

## Phase tracking

In this project two scenarios were looked at. One where no phase tracking was one, and one where the Viterbi-Viterbi algorithm was used.   
The Viterbi-Viterbi algorithm is used for phase tracking and equalization of received OFDM symbols, ensuring accurate detection despite phase distortions caused by the channel. Initially, a known training symbol is used to estimate the phase shift introduced by the channel. This phase shift is calculated by comparing the received signal with the training symbol. As the receiver processes subsequent symbols, the algorithm tracks changes in the phase by estimating the current phase shift and comparing it to the previous one. A set of possible phase shifts is used to find the closest match, minimizing errors caused by phase drift. The phase estimate is then smoothed to ensure stability, combining the current and previous estimates. Finally, the phase correction is applied to the received symbols, compensating for any phase misalignments, and the equalized symbols are computed. This approach effectively mitigates phase-related distortions and enhances the accuracy of symbol detection, particularly in the presence of phase noise or Doppler shifts.

## Channel equalization

Two different channel equalization methods are implemented, based on the type of training used: one using a conventional training symbol for each OFDM symbol and the other using a comb-type training pattern. In the first method, the channel frequency response is estimated by dividing the received training symbols by the known reference training symbol. This estimate is then used to equalize the received data symbols by compensating for the channel effects, allowing for improved symbol detection.

The second method, called comb-type training, inserts known training symbols at specific intervals across the carriers. For each OFDM symbol, the training positions are identified, and the channel is estimated for these positions by dividing the received symbols by the reference training symbols. The channel estimate is then used to equalize all subsequent symbols for those carriers, allowing for better recovery of the payload data. The comb-type training method is particularly useful in scenarios where fewer training symbols are available but can still provide reliable channel estimates for the carriers with training symbols, helping to reduce the impact of channel distortions on the received signal.

## Parallel to Serial conversion

After the channel equalization, the next step involves converting the received data from parallel to serial form. This is achieved by the serial-to-parallel conversion function, which reorganizes the serial data into multiple parallel streams, corresponding to the individual subcarriers used in the OFDM system. In this process, the data is reshaped so that each column represents the samples for a specific symbol and subcarrier. The reshaped matrix has dimensions based on the number of subcarriers and the number of symbols, allowing for the alignment of the data for further processing.

## Normalizing signal energy

The signal energy is normalized to ensure consistent power levels across the transmitted symbols, preventing any potential distortion or imbalance during the signal processing stages.

## Demapping

The demapping process converts the received symbols back into bits by checking the real and imaginary parts of each symbol. For each symbol, the real part determines the first bit, and the imaginary part determines the second bit. The result is a binary vector, with each element representing the corresponding bit value from the symbol. This step is crucial for recovering the transmitted data from the modulated symbols.

After all this processing of the signal is done, the receiver side has the RX bits which can be shown via various plots and analysing of the channel and flow can be done.

A screen shot of a graph

Description automatically generated

Figure 8 Spectrum of received signal

A screen shot of a sound wave

Description automatically generated

Figure 9 Signal that was received'

# Analysis

## Channel analysis

The average channel frequency response was examined to understand how the channel affected the transmitted signal across different subcarriers. The plot of the average channel frequency response provides insights into the variations in channel gain over the frequency spectrum, highlighting the frequency-dependent characteristics of the channel. This information is crucial for evaluating channel conditions and optimizing the equalization process. The plot was obtained by dividing the spectrum of the received training symbols by the spectrum of the known ones.

A graph showing a graph of a graph

Description automatically generated with medium confidence

Figure 10 Average channel frequency response

The channel impulse response provides valuable insights into how the real channel impacts the transmitted signal. By performing an inverse Fourier Transform of the estimated channel , the channel impulse response was obtained. This response illustrates the time-domain characteristics of the channel, including its delay spread. Most of the energy was concentrated within the first few milliseconds, indicating that the channel's influence is primarily localized within this time span. This information is crucial for determining the minimum cyclic prefix length required to mitigate ISI while maintaining system efficiency.

To discuss the minimum length of the CP, so that ISI is avoided on the received side, the Channel Impulse Response needs to be looked at. With a sampling frequency of  
 , the minimum number of taps is 33, which was determined by counting the number of samples in the delay spread.

A graph with blue lines

Description automatically generated

Figure 11 Channel impulse response

Another aspect was inspected, that being how did the CP length affect on the correlation between BER and Spectral Efficiency.

Spectral Efficiency is defined as:

Where N is the number of subcarriers and is the length of CP. Accordingly the longer the CP, the lower the spectral efficiency of the system. In this particular project, the spectral efficiency measured in a big room was 66.7%.

Noticeably from Figure 12, it can be seen that Comb type of training symbols have lowest BER for high spectral efficiency.

This is because Comb training interleaves training symbols across subcarriers, providing frequent and precise channel estimates that capture the channel's frequency-selective nature. The reduced training overhead also ensures a higher spectral efficiency, as more subcarriers are available for data transmission. Additionally, Comb training is robust against rapid channel variations, maintaining accuracy even in challenging conditions. These advantages make Comb-type training the optimal choice for balancing performance and efficiency, as reflected in the results.

A graph of a graph with different colored lines

Description automatically generated with medium confidence

Figure 12 BER vs. Spectral Efficiency based on Ncp

Next was looked at the dependency between the BER in terms of different tracking methods. From Figure 13 it can be notices that in each case the higher the number of carriers the bigger the BER was, which is understandable. This outcome is understandable because as the number of carriers increases, the subcarrier spacing decreases for a fixed bandwidth. This narrower spacing makes the system more sensitive to inter-carrier interference (ICI) and frequency-selective fading, especially in channels with significant delay spread or Doppler shifts. Additionally, the increased number of carriers requires more precise synchronization and channel estimation, which can be challenging to achieve in practical systems, thereby contributing to a higher BER. In this case, Comb showed the best results as well.

A graph showing different colored lines

Description automatically generated

Figure 13 BER vs. Number of Carriers

We can get to the same conclusion with Figure 14 as well. Taking the relationship between the BER and the spacing between carriers with fixed bandwidth, the lower the spacing, the higher the BER. This is because narrower spacing makes the system more prone to intercarrier interference (ICI) due to Doppler effects or synchronization errors. Additionally, in channels with significant delay spread, insufficient spacing can also lead to overlapping signals, contributing to ISI if the cyclic prefix is not adequately designed. And the statement that Comb performs best is also supported.

A graph of different colored lines

Description automatically generated

Figure 14 BER vs. Spacing between carriers

The system was put in 4 different scenarios: in a big room, outdoors, in a small room  
(EPFL locker) and while moving. The goal was to see how the space and physical conditions impact the state of the response of the system. The taps seen in the Impulse Response represent the reflexions of the channel. Figure 15 shows the channel response in a small room, EPFL locker, with noticeable reflexions because of the small distance and space the signal transmits in. Figure 16 shows the system in bigger room, and the spikes are visibly lower because the space in which the signal propagates is bigger. On Figure 17 the spikes are nearly non existent because the system is out outdoors. The last case, Figure 18 shows the system in a moving condition. The spikes are strong and dynamic due to constantly changing channel conditions, which introduce varying levels of reflections and distortions.

A graph with blue lines

Description automatically generated

Figure 15 Channel Impulse response in small room

A graph with blue lines

Description automatically generated

Figure 16 Channel Impulse response in big room

A graph with numbers and lines

Description automatically generated

Figure 17 Channel Impulse response outdoors

A screen shot of a graph

Description automatically generated

Figure 18 Channel Impulse response from moving system

Another thing that can be examined is the performance degradation in terms of implementing phase tracking or not. This was look at with a steady and moving system. In both cases Viterbi showed better results in regarding lower BER, which was already expected.

A graph of a graph with blue and orange lines

Description automatically generated

Figure 19 Performance degradation in a moving system

A graph with blue and orange lines

Description automatically generated

Figure 20 Performance degradation in a steady system

Last consideration looked at was the relationship between the training rate and BER. Inserting more training symbols slowed the transmission but guaranteed higher precision while transmitting. So, it is crucial to find the perfect balance to get optimal result. Worth mentioning is that Viterbi tracking showed lowest BER with the lowest training rate.

A graph of different training rate

Description automatically generated with medium confidence

Figure 21 Training rate vs. BER

# Image transmitting and receiving

An additional objective of this project was to evaluate the system's performance in transmitting and receiving an image. This task involved encoding the image into binary data, transmitting it over the simulated communication channel, and reconstructing the image at the receiver side. The process included:

1. **Image Encoding**: The original image was converted into a binary stream by reshaping the image into a bit matrix, where each pixel value was represented as 8 bits. This binary stream was then prepared for transmission through the QPSK-based OFDM system.
2. **Transmission and Channel Effects**: The binary stream was modulated and transmitted over the channel, where it experienced noise, fading, and other channel impairments. The communication system introduced potential distortions to the received signal.
3. **Decoding and Reconstruction**: On the receiver side, the transmitted binary stream was demodulated, equalized, and decoded. The received bits were mapped back into pixel values and reshaped into the dimensions of the original image, effectively reconstructing the transmitted image.

Figure 22 shows the comparison between the original and the reconstructed image after passing through the entire process. The quality of the received image reflects how well the system handled the effects of noise, channel distortions, and synchronization issues.

With this transmission we managed to get BER of .

A comparison of a horse's face

Description automatically generated

Figure 22 Transmission and receiving the image

# Conclusion

This project focused on evaluating the performance of an OFDM-based communication system with QPSK modulation, under varying conditions. A thorough analysis was conducted on different aspects of the system, including channel estimation methods, cyclic prefix (CP) length, and the impact of environmental factors on the channel response.

The results demonstrated that the Comb-type training method, with its ability to efficiently handle noise and fast channel variations, provided the best performance under challenging conditions. This is especially true for scenarios where the channel state is unstable, such as in outdoor or moving environments. On the other hand, the Block training method proved effective for more stable channels, where phase variations were slower, showing a robust performance with lower BER.

The study also highlighted the importance of selecting an optimal CP length based on the channel's delay spread. By analyzing the channel impulse response, we determined the minimum CP length required to mitigate Inter-Symbol Interference (ISI), ensuring signal integrity during transmission. This finding is crucial for optimizing the efficiency of the system without sacrificing reliability.

Additionally, the evaluation of system performance in transmitting and receiving images demonstrated the robustness of the OFDM system in handling real-world data, even under imperfect channel conditions. The reconstructed image, despite being affected by noise and impairments, reflected the effectiveness of the system's error correction and equalization processes.

In conclusion, the implemented OFDM system showed versatility in adapting to various channel conditions, with strong potential for real-time data transmission applications. The system’s robustness, efficiency, and flexibility underscore the viability of OFDM modulation as a foundational technology for modern wireless communication systems.

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